Homework 7

Goal: To learn to numerically solve ODEs in Matlab using the `ode45()` function and `events`.

Due: April 30

Problems:

1. ***Relativistic Spring (part 2).*** Modify your relativistic spring problem (HW6, P2) to do the following:

   (a) Add a third graph to your HW6 plot showing the total relativistic energy of the system over time:

   $$E = \frac{1}{2}kx^2 + \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

   (b) Overlay multiple curves on the same plot showing different initial displacements x_0. Pick 4 or 5 initial values of x_0 that clearly show the transition from Newtonian to Relativistic motion. Create a legend.

   (c) Use events to find the maximum speed of the particle. Mark this location on both the position-vs-time plot and the velocity-vs-time plot.

   (d) Use events to find the maximum spring elongation (i.e. max and min of the particle position). Mark this location using a different symbol on both the position-vs-time plot and the velocity-vs-time plot.

   (e) Use the fprintf() command to output a table on the command line. The table should be nicely formatted and show:

   - initial displacement
   - maximum velocity predicted by Newton (as a fraction of the speed of light)
   - maximum velocity (as a fraction of the speed of light)
   - period (as measured by event pairs)
   - total energy

   (f) Is energy conserved in this system? Is the period independent of the initial displacement?

   What to turn in: P1.m, P1.pdf

2. ***Relativistic Spring (part 3).***

   (a) Plot the period of the relativistic spring as a function of the initial displacement. Use at least 100 initial displacement values. Overlay a horizontal line showing the classical, Newtonian prediction for the spring's period. Pick a range of initial displacements so the transition from Newtonian to Relativistic motion is clear.

   (b) Plot the maximum velocity of the relativistic spring as a function of the initial displacement. Use at least 100 initial displacement values. Overlay a curve showing the classical, Newtonian prediction for the spring's maximum velocity. Pick a range of initial displacements so the transition from Newtonian to Relativistic motion is clear.

   What to turn in: P2.m, P2.pdf