Matlab (Matrix Laboratory)

5 Integrated Parts:

- Programming language
- Working environment
- Graphics
- Mathematical function library (organized by toolboxes):
  - curve fitting toolbox
  - optimization toolbox
  - partial differential equation (PDE) toolbox
  - …
- Application Program Interface (API) - lets you write programs in other languages (C, Fortran, etc) if you like.

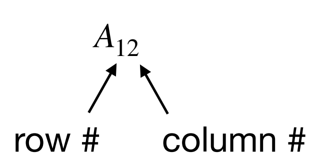
Matrices

An $n \times m$ matrix is a rectangular group of numbers with *n* rows and *m* columns. For example, the following matrix is a 3×4 matrix:

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 7 & 5 \\ 8 & 5 & 11 & -4 \\ 3 & 2 & 6 & -1 \end{pmatrix}$$

The elements of a matrix are labeled by subscripts. For example $A_{12} = -2$ in the above example. Each element is labeled in the following matrix:

$$\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \end{pmatrix}$$



Scalars and Vectors

Scalars and vectors are special types of matrices:

1×1 matrix = scalar $\quad \mathbf{A} = (4)$

$n \times 1$ matrix = column vector $\quad \mathbf{A} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$

$1 \times m$ matrix = row vector $\quad \mathbf{A} = (6 \quad 7 \quad 3)$

Two common uses of row or column vectors

1. Store components of position, velocity, etc.:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

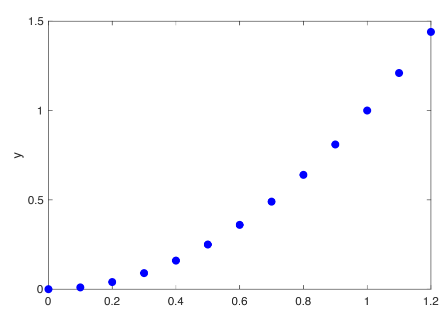
$$\vec{r} = 2\hat{i} + 3\hat{j} - 1\hat{k} \;\rightarrow\; \mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \quad \text{or} \quad \mathbf{A} = (2 \quad 3 \quad -1)$$

2. Store data or simulated data sampled in time or space

example:

$$\mathbf{x} = (0.00 \quad 0.10 \quad 0.20 \quad 0.30 \quad 0.40 \quad \ldots)$$

$$\mathbf{y} = (0.00 \quad 0.01 \quad 0.04 \quad 0.09 \quad 0.16 \quad \ldots)$$



Transpose

The transpose of a matrix is obtained by ``flipping'' it around the diagonal. The symbol $^\intercal$ is called the transpose operator.

$$\mathbf{A}^\intercal_{ij} = \mathbf{A}_{ji}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}^\intercal = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

Transpose of a row vector gives a column vector

$$(1 \quad 2 \quad 3)^\intercal = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Transpose of a column vector gives a row vector

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}^\intercal = (1 \quad 2 \quad 3)$$

Adjoint

The adjoint of a matrix is equal to the complex conjugate of the transpose. The adjoint operator is represented by †.

$$\mathbf{A}^\dagger_{ij} = \mathbf{A}^*_{ji}$$

$$\begin{pmatrix} 1 & 2i & 0 \\ 1+i & i & 6 \\ 7 & 0 & 0 \end{pmatrix}^\dagger = \begin{pmatrix} 1 & 1-i & 7 \\ -2i & -i & 0 \\ 0 & 6 & 0 \end{pmatrix}$$

Adjoint of a row vector gives a column vector

$$(1 \quad 2 \quad 3i)^\dagger = \begin{pmatrix} 1 \\ 2 \\ -3i \end{pmatrix}$$

Adjoint of a column vector gives a row vector

$$\begin{pmatrix} 1 \\ 2 \\ -3i \end{pmatrix}^\dagger = (1 \quad 2 \quad 3i)$$

Addition and Multiplication of Matrices

Matrix Addition

Matrices must have the same numbers of rows and columns to be added. The matrices are added element-by-element. The result is:

$$\mathbf{A}_{ij} + \mathbf{B}_{ij} = \mathbf{C}_{ij}$$

Example:

$$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$$

Example:

$$\begin{pmatrix} 2 & 4 & 6 \end{pmatrix} + \begin{pmatrix} 1 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 3 & 4 & 4 \end{pmatrix}$$

Matrix Multiplication

In order to multiply two matrices, the number of columns of the first matrix must equal the number of rows of the second matrix. The elements of each row of the first matrix are multiplied by each column in the second matrix (and summed):

$$C_{ij} = \sum_k A_{ik} \cdot B_{kj}$$

Example:

$$\begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 3 & 4 \\ 4 & -1 \end{pmatrix} =$$

Multiplication of Row × Column Vector

This case is equivalent to the inner product (or "dot" product) of 2 vectors. The result is a scalar.

$$C_{11} = \sum_k A_{1k} \cdot B_{k1}$$

Note: each vector must have the same number of elements.

Example: $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 20$

Multiplication of Column × Row Vector

Multiplication of a $n \times 1$ column vector by a $1 \times m$ row vector produces a $n \times m$ matrix called the outer product: †

$$C_{nm} = A_{n1} \cdot B_{1m}$$

Example:

$$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} (1 \quad 2 \quad 3) = \begin{pmatrix} 2 & 4 & 6 \\ 3 & 6 & 9 \\ 4 & 8 & 12 \end{pmatrix}$$