Galilean Transformation

$$x' = x - ut \qquad v'_x = v_x - u$$
$$y' = y \qquad v'_y = v_y$$
$$z' = z \qquad v'_z = v_z$$
$$t' = t$$

$$\vec{v}' = \vec{v} - \vec{u}.$$

Lorentz Transformations:

$$x' = \gamma(x - ut) \qquad x = \gamma(x' + ut')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$
$$t' = \gamma\left[t - \left(\frac{u}{c^2}\right)x\right] \qquad t = \gamma\left[t' + \left(\frac{u}{c^2}\right)x'\right]$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$$\Delta x' = \gamma(\Delta x - u\Delta t) \qquad \Delta x = \gamma(\Delta x' + u\Delta t')$$
$$\Delta y' = \Delta y \qquad \Delta y = \Delta y'$$
$$\Delta z' = \Delta z \qquad \Delta z = \Delta z'$$
$$\Delta t' = \gamma\left[\Delta t - \left(\frac{u}{c^2}\right)\Delta x\right] \quad \Delta t = \gamma\left[\Delta t' + \left(\frac{u}{c^2}\right)\Delta x'\right]$$

Velocity Transformations

$$v'_x = \frac{v_x - u}{1 - v_x u/c^2} \qquad v_x = \frac{v'_x + u}{1 + v'_x u/c^2}$$
$$v'_y = \frac{v_y\sqrt{1 - u^2/c^2}}{1 - v_x u/c^2} \qquad v_y = \frac{v'_y\sqrt{1 - u^2/c^2}}{1 + v'_x u/c^2}$$
$$v'_z = \frac{v_z\sqrt{1 - u^2/c^2}}{1 - v_x u/c^2} \qquad v_z = \frac{v'_z\sqrt{1 - u^2/c^2}}{1 + v'_x u/c^2}$$

Time Dilation

$$\tau = \gamma\tau_0$$

Length Contraction

$$L = L_0/\gamma$$

$$u = \frac{L_0}{\tau} \qquad u = \frac{L}{\tau_0}$$

Doppler Effect

$$f' = f\sqrt{\frac{1 - u/c}{1 + u/c}}$$

Relativistic Momentum

$$\vec{\mathbf{p}} = \gamma(v)m\vec{\mathbf{v}}$$

$$\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Force

$$F = \frac{dp}{dt}$$

Relativistic Kinetic Energy

$$K = (\gamma(v) - 1)mc^2$$

Total Energy

$$E = E_0 + K$$

$$E = \gamma(v)mc^2$$

Rest Energy

$$E_0 = mc^2$$

$$E^2 = (pc)^2 + (mc^2)^2$$

Massless Particle

$$E = pc$$

Nonrelativistic Particle ($K \ll E_0$ or $v \ll c$)

Highly Relativistic Particle ($K \gg E_0$)

$$E \approx pc$$

Light as a Particle

Photon Energy and Momentum

$$E = hf$$

$$p = \frac{h}{\lambda} = \frac{hc}{\lambda}\frac{1}{c}$$

Photoelectric Effect

$$K_{\text{max}} = hf - \phi$$

$$\lambda_c = \frac{hc}{\phi}$$

Thermal Radiation

$$P = IA$$

$$I = \sigma T^4$$

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3}\text{ m K}}{T}$$

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

Compton Scattering

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\lambda_{\text{Compton}} = \frac{h}{m_e c} = 2.426\text{ pm}$$

Particles as Waves

$$E = hf \qquad E = \hbar\omega$$

$$p = \frac{h}{\lambda} \qquad p = \hbar k$$

Phase Velocity

$$v_p = \frac{\omega}{k} = \lambda f$$

Group Velocity

$$v_g = \frac{d\omega}{dk}$$

Superposition of Two Waves

$$v_p = \frac{\omega_1 + \omega_2}{k_1 + k_2} \qquad v_g = \frac{\omega_2 - \omega_1}{k_2 - k_1}$$

Uncertainty Principles

$$\Delta x \Delta p \geq \frac{\hbar}{2} \qquad \Delta t \Delta E \geq \frac{\hbar}{2}$$

Rutherford Scattering

Distance of Closest Approach

$$d = \frac{1}{4\pi\epsilon_0}\frac{zZe^2}{K}$$

Bohr Model of the Atom

Quantized Angular Momentum

$$L_n = mvr = n\hbar$$

Orbital Radius

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2}n^2 = a_0 n^2 \qquad a_0 = 0.0529 \text{ nm}$$

Energy Levels

$$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = -\frac{13.6 \text{ eV}}{n^2}$$

Transition energies

$$hf = E_u - E_l = (13.6 \text{ eV})\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

de Broglie Waves

$$E = hf \qquad E = \hbar\omega$$

$$p = \frac{h}{\lambda} \qquad E = \hbar k$$

1D Schrodinger Equation

Time-Dependent Schrodinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Time-Independent Schrodinger Equation

$$\Psi(x,t) = \psi(x)e^{-i\omega t}$$

$$\frac{\partial^2\psi(x)}{\partial x^2} = -\frac{2m}{\hbar^2}\left[E - U(x)\right]\psi(x)$$

Normalization Condition

$$\int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = 1$$

Probability on small interval Δx

$$P(x) = |\Psi(x)|^2 \Delta x$$

Probability on interval (a, b)

$$P(a,b) = \int_a^b |\Psi(x)|^2 dx$$

Constant Potential Solutions $U(x) = U_0$

$E > U_0$

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

or

$$\psi(x) = A'e^{ikx} + B'e^{-ikx}$$

$$k = \frac{\sqrt{2m(E - U_0)}}{\hbar}$$

$E < U_0$

$$\psi(x) = Ce^{k'x} + De^{-k'x}$$

$$k' = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

Infinite Potential Well (Particle in a Box)

$$U(x) = \begin{cases} +\infty & \text{for } x \leq 0 \\ 0 & \text{for } 0 < x < L \\ +\infty & \text{for } x \geq L \end{cases}$$

Standing Wave Solutions

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \qquad n = 1, 2, 3, ...$$

Energy States

$$E_n = \frac{h^2 n^2}{8mL^2} \qquad n = 1, 2, 3, ...$$

Step Potential $(E > U_0)$

$$U(x) = \begin{cases} 0 & \text{for } x \leq 0 \qquad \text{(Region I)} \\ U_0 & \text{for } 0 \qquad \text{(Region II)} \end{cases}$$

Reflection Probability

$$R = \frac{|B'|^2}{|A'|^2} = \left[\frac{1 - k_2/k_1}{1 + k_2/k_1}\right]^2$$

Transmission Probability

$$T = \frac{|C'|^2}{|A'|^2} = \frac{4(k_2/k_1)^2}{(1 + k_2/k_1)^2}.$$

Step Barrier $(E < U_0)$

$$U(x) = \begin{cases} 0 & \text{for } x \leq 0 \qquad \text{(Region I)} \\ U_0 & \text{for } 0 < x \leq L \qquad \text{(Region II)} \\ 0 & \text{for } x > L \qquad \text{(Region III)} \end{cases}$$

Tunneling Probability

$$T \approx \frac{16E(U_0 - E)}{U_0^2} e^{-2k'L}$$

2D Schrodinger Equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}) + U(\vec{r})\psi(\vec{r}) = E\psi(\vec{r}),$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$$

Infinite Potential Well in 2D

$$U(x, y) = \begin{cases} 0 & \text{for } 0 < x < L;\ 0 < y < L \\ +\infty & \text{otherwise} \end{cases}$$

Wave Functions

$$\psi(x, y) = \frac{2}{L} \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right) \qquad n_x, n_y = 1, 2, 3, ...$$

Energy Levels

$$E_{n_x,n_y} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2\right) \qquad n_x, n_y = 1, 2, 3, ...$$

Spin
Magnetic moment

$$\vec{\mu} = -\frac{e}{2m_e}\vec{\mathbf{S}}.$$

$$\mu_z - m_s\,\mu_B$$

Bohr magneton

$$\mu_B = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ J/T}.$$

Hydrogen Quantum Numbers:

	Orbital	Spin
Quantum number	$l = 0, 1, 2, 3, ...$	$s = 1/2$
Magnitude of angular momentum vector	$\lvert\vec{\mathbf{L}}\rvert = \sqrt{l(l+1)}\,\hbar$	$\lvert\vec{\mathbf{S}}\rvert = \sqrt{s(s+1)}\,\hbar$
z component of angular momentum	$L_z = m_l\hbar$	$S_z = m_s\hbar$
magnetic quantum number	$m_l = 0, \pm1, \pm2, ..., \pm l$	$m_s = \pm 1/2$

Masses and Constants

	Quantity	MKS		eV units	
c	speed of light	2.9979×10^{8}	m/s		
e	fundamental charge	1.602×10^{-19}	C		
eV	electron volt	1.602×10^{-19}	J		
h	Planck's constant	6.626×10^{-34}	J s	4.136×10^{-15}	eV s
$\hbar$	"h bar"	1.055×10^{-34}	J s	6.582×10^{-16}	eV s
hc		1.986×10^{-25}	J m	1240	eV nm (or MeV fm)
k	Boltzmann's constant	1.381×10^{-31}	J/K	8.617×10^{5}	eV / K
σ	Stefan-Boltzmann constant	5.670×10^{-8}	W m^{-2} K^{-4}		
e^{-}	mass of electron	9.11×10^{-31}	kg	0.511	MeV/c^2
p	mass ofproton	1.673×10^{-27}	kg	938.3	MeV/c^2
n	mass of neutron	1.675×10^{-27}	kg	939.6	MeV/c^2

1 μm	$= 10^{-6}$ m	1 keV	$= 10^{3}$ eV
1 nm	$= 10^{-9}$ m	1 MeV	$= 10^{6}$ eV
1 pm	$= 10^{-12}$ m	1 GeV	$= 10^{9}$ eV
1 fm	$= 10^{-15}$ m	1 TeV	$= 10^{12}$ eV

Math

Binomial Expansion

$$(1-x)^n \approx 1 - nx \qquad x \ll 1$$

Complex Numbers

Definitions

$$z = a + bi \qquad z^* = a - bi$$

$$z = |z|e^{i\phi} \qquad z^* = |z|e^{-i\phi}$$

Modulus squared

$$|z|^2 = z^* z = a^2 + b^2$$

Euler's Formula

$$e^{i\phi} = \cos\phi + i\sin\phi$$

Identities

$$\cos\phi = \frac{1}{2}(e^{i\phi} + e^{-i\phi})$$

$$\sin\phi = \frac{1}{2i}(e^{i\phi} - e^{-i\phi})$$

Complex Traveling Wave

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

Complex Standing Wave

$$\Psi(x,t) = A\cos(kx)e^{-i\omega t}$$