Complex Numbers

Definitions

$$z = a + bi \qquad z^* = a - bi$$

$$z = |z|e^{i\phi} \qquad z^* = |z|e^{-i\phi}$$

Modulus squared

$$|z|^2 = z^* z = a^2 + b^2$$

Euler's Formula

$$e^{i\phi} = \cos\phi + i\sin\phi$$

Identities

$$\cos\phi = \frac{1}{2}(e^{i\phi} + e^{-i\phi})$$

$$\sin\phi = \frac{1}{2i}(e^{i\phi} - e^{-i\phi})$$

Complex Traveling Wave

$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

Complex Standing Wave

$$\Psi(x,t) = A\cos(kx)e^{-i\omega t}$$

de Broglie Waves

$$E = hf \qquad E = \hbar\omega$$

$$p = \frac{h}{\lambda} \qquad E = \hbar k$$

1D Schrodinger Equation

Time-Dependent Schrodinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Time-Independent Schrodinger Equation

$$\Psi(x,t) = \psi(x)e^{-i\omega t}$$

$$\frac{\partial^2\psi(x)}{\partial x^2} = -\frac{2m}{\hbar^2}\left[E - U(x)\right]\psi(x)$$

Normalization Condition

$$\int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = 1$$

Probability on small interval Δx

$$P(x) = |\Psi(x)|^2\Delta x$$

Probability on interval (a,b)

$$P(a,b) = \int_a^b |\Psi(x)|^2 dx$$

Constant Potential Solutions $U(x) = U_0$

$E > U_0$

$$\psi(x) = A\sin(kx) + B\cos(kx)$$

or

$$\psi(x) = A'e^{ikx} + B'e^{-ikx}$$

$$k = \frac{\sqrt{2m(E-U_0)}}{\hbar}$$

$E < U_0$

$$\psi(x) = Ce^{k'x} + De^{-k'x}$$

$$k' = \frac{\sqrt{2m(U_0-E)}}{\hbar}$$

Infinite Potential Well (Particle in a Box)

$$U(x) = \begin{cases} +\infty & \text{for } x \le 0 \\ 0 & \text{for } 0 < x < L \\ +\infty & \text{for } x \ge L \end{cases}$$

Standing Wave Solutions

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \qquad n = 1, 2, 3, ...$$

Energy States

$$E_n = \frac{h^2 n^2}{8mL^2} \qquad n = 1, 2, 3, ...$$

Step Potential $(E > U_0)$

$$U(x) = \begin{cases} 0 & \text{for } x \le 0 & \text{(Region I)} \\ U_0 & \text{for } 0 & \text{(Region II)} \end{cases}$$

Reflection Probability

$$R = \frac{|B'|^2}{|A'|^2} = \left[\frac{1 - k_2/k_1}{1 + k_2/k_1}\right]^2$$

Transmission Probability

$$T = \frac{|C'|^2}{|A'|^2} = \frac{4k_2/k_1}{(1 + k_2/k_1)^2}.$$

Step Barrier $(E < U_0)$

$$U(x) = \begin{cases} 0 & \text{for } x \le 0 & \text{(Region I)} \\ U_0 & \text{for } 0 < x \le L & \text{(Region II)} \\ 0 & \text{for } x > L & \text{(Region III)} \end{cases}$$

Tunneling Probability

$$T \approx \frac{16E(U_0 - E)}{U_0^2}e^{-2k'L}$$

2D Schrodinger Equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}) + U(\vec{r})\psi(\vec{r}) = E\psi(\vec{r}),$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}$$

Infinite Potential Well in 2D

$$U(x,y) = \begin{cases} 0 & \text{for } 0 < x < L;\ 0 < y < L \\ +\infty & \text{otherwise} \end{cases}$$

Wave Functions

$$\psi(x,y) = \frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right) \qquad n_x, n_y = 1, 2, 3, ...$$

Energy Levels

$$E_{n_x,n_y} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2\right) \qquad n_x, n_y = 1, 2, 3, ...$$

Masses and Constants

	Quantity	MKS		Other			
c	speed of light	2.9979×10^8	m/s				
e	fundamental charge	1.602×10^{-19}	C				
eV	electron volt	1.602×10^{-19}	J				
h	Planck's constant	6.626×10^{-34}	J s	4.136×10^{-15}	eV s		
$\hbar$	"h bar"	1.055×10^{-34}	J s	6.582×10^{-16}	eV s		
hc		1.986×10^{-25}	J m	1240	eV nm	1240	MeV fm
k	Boltzmann's constant	1.381×10^{-31}	J/K	8.617×10^{5}	eV / K		
σ	Stefan-Boltzmann constant	5.670×10^{-8}	W m^{-2} K^{-4}				
e^-	mass of electron	9.11×10^{-31}	kg	0.511	MeV/c^2		
p	mass ofproton	1.673×10^{-27}	kg	938.3	MeV/c^2		
n	mass of neutron	1.675×10^{-27}	kg	939.6	MeV/c^2		

1 μm	$= 10^{-6}$ m	1 keV	$= 10^3$ eV
1 nm	$= 10^{-9}$ m	1 MeV	$= 10^6$ eV
1 pm	$= 10^{-12}$ m	1 GeV	$= 10^9$ eV
1 fm	$= 10^{-15}$ m	1 TeV	$= 10^{12}$ eV