# Lecture 7. Light as a Particle

## Contents




**Summary.** Evidence for the wave nature of light slowly mounted over the course of the 19th Century, culminating in Maxwell's theory of electromagnetism. At the beginning of the 20th Century, Max Planck and Albert Einstein suggested light also acts like a particle in some situations. This lecture reviews evidence for the wave-nature of light and then explores the revolutionary ideas proposed by Planck and Einstein.


## 7.1 Evidence of the Wave Nature of Light

As Maxwell showed in the 1860s, light may be described as an electromagnetic wave. The solution of an electromagnetic plane wave traveling in the $z$ direction is given by

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 \sin(kz - \omega t) \qquad \vec{\mathbf{B}} = \vec{\mathbf{B}}_0 \cos(kz - \omega t) \tag{7.10}$$

where the wave number $k$ is found from the wavelength $\lambda$ through $k = 2\pi/\lambda$ and the angular frequency $\omega$ is found from the frequency $f$ from $\omega = 2\pi f$. The vector product $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ points in the direction of travel.

The radiative intensity $I$ of the wave is the power per unit area averaged over one oscillation period. It is proportional to the square of the amplitude of the electric or magnetic field and is given by

$$I = \frac{P_{ave}}{A} = \frac{1}{2\mu_0 c} E_0^2. \tag{7.11}$$

The units of radiative intensity are $\mathrm{W/m^2}$. The **monochromatic intensity** is defined as the intensity in a wavelength interval $d\lambda$ and is written $I(\lambda)d\lambda$. The total intensity across all wavelengths is sometimes called the **bolometric intensity** and is given by

$$I = \int_0^\infty I(\lambda) d\lambda.$$

Evidence of the wave-nature of light is provided by the following experiments:

1. **Young's Double-Slit Experiment.** A plane wave passing through two closely spaced, parallel slits produces a pattern of constructive and destructive interference fringes on a distant screen if the the spacing and with of the slits are comparable to the wavelength of the light. If the path length between a point on the screen and the two slits is an integer number $n$ of wavelengths $\lambda$ then a bright fringe signaling **constructive interference** will appear:

$$|s_1 - s_2| = n\lambda.$$

If the path length between a point on the screen and the two slits differs by a half a wavelength then a dark fringe signaling **destructive interference** will appear:

$$|s_1 - s_2| = \left(n + \frac{1}{2}\right)\lambda.$$

Let's define the perpendicular distance from the screen to the slits to be $D$ and the separation of the slits to be $d$. The distance from the center of the central bright fringe to the $n^{th}$ bright fringe is

$$y_n = n\frac{\lambda D}{d}.$$

2. **Crystal Diffraction of X-Rays.** The regular spacing of atoms in a crystal may be used as slits for creating diffraction patterns if the wavelength of the light is comparable to the atomic spacing, which is a fraction of a nanometer. This wavelength lies in the X-ray portion of the electromagnetic spectrum. The X-ray diffraction pattern created by crystals is a clear indication that X-rays exhibit wave phenomena.

## 7.2 Thermal Radiation

All sufficiently condensed objects (planets, stars, coals in a fire, ice cubes, people, etc.) give off radiation due to the thermal motion of their atoms. At room temperature most of this radiation is emitted in the infrared part of the spectrum and is invisible to our eyes. Objects (like the sun) that are a fer thousand degrees K emit visible light.

We consider an idealized object that is a *perfect* radiator. Such an object is called a **blackbody** because it absorbs all radiation that falls on it, i.e. it looks black if it is sufficiently cold. If it is heated to a temperature $T$, then it will radiate according to the following two laws:

1. **Stefan-Boltzmann Law** The total intensity (power per unit area) radiated by a blackbody over all wavelengths is proportional to the object's temperature raised to the fourth power:

$$I = \sigma T^4$$

where $\sigma = 5.67 \times 10^{-8}$ W/m$^2$ K$^4$.

The energy density (energy per unit volume) of isotropic radiation in a blackbody is given by

$$u = \frac{4\sigma}{c}T^4.$$

**Example 1.** Find the total power radiated by a perfectly radiating sphere with radius 10 cm at room temperature (293 K).

**Solution.** The total power is

$$\begin{aligned} P = IA &= (4\pi r^2)\sigma T^4 \\ &= \left(4\pi(0.1m)^2\right)\left(5.67\times 10^{-8}\right) 293^4 \\ &= 52.5 \text{ W} \end{aligned}$$

2. **Wien's Displacement Law:** The wavelength $\lambda_{max}$ where the intensity of the thermal radiation is greatest is inversely proportional to the object's temperature:

$$\lambda_{max} = \frac{2.898 \times 10^{-3} \text{ m K}}{T}$$

For visible light photons is is sometimes convenient to express the constant in nm instead of m:

$$\lambda_{max}(\text{in nm}) = \frac{2.898 \times 10^{6} \text{ nm K}}{T}$$

**Example 2.** Consider two stars with the same radii, but with different temperatures. Star A has a temperature of 10,000 K and star B has a temperature of 3,000 K. Find (a) the relative power radiated the stars and (b) the wavelength of the peak intensity of each star.

**Solution.** (a) The ratio of the total total power is

$$\frac{P_A}{P_B} = \frac{I_A A}{I_B A} = \left(\frac{T_A}{T_B}\right)^4 = \left(\frac{10,000}{3,000}\right)^4 = 123.5$$

(b) The peak intensity of star A is

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{6} \text{ nm K}}{10,000 \text{ K}} = 289.8 \text{ nm}$$

which is in the ultraviolet.

The peak intensity of star B is

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{6} \text{ nm K}}{3,000 \text{ K}} = 966 \text{ nm}$$

which is in the infrared.

### 7.2.1 Classical Theory of Thermal Radiation

Toward the end of the 19th Century Lord Rayleigh applied classical thermodynamics to describe radiation fields. He treated radiation in a cavity like a system of molecules in a box. Individual molecules were replaced by electromagnetic standing waves. The energy density $u$ (energy per unit volume) of the system may be written as

$$\text{energy density} = \left(\frac{\text{energy}}{\text{vol.}}\right) = \left(\frac{\#\ \text{oscillators}}{\text{vol.}}\right)\left(\frac{\text{ave. energy}}{\text{oscillator}}\right).$$

In symbols this becomes

$$u = n \cdot \bar{E},$$

where $n$ is the number of oscillators per unit volume and $\bar{E}$ is the average energy per oscillator.

The number of standing waves in a three-dimensional unit box is given by

$$n = \frac{8\pi}{\lambda^4}. \tag{7.12}$$

As the wavelength of the standing wave decreases, the number of waves per unit volume increases sharply as $\lambda^{-4}$. In classical physics the energy contained in the standing wave is proportional to the amplitude squared and can take on any value, i.e. it is a continuously distributed variable.

In 1877, Ludwig Boltzmann derived an expression for the probability of finding a system in an energy state $E$. The distribution function $f(E)$ is given by

$$f(E) \propto e^{-E/kT}$$

where $k$ is Boltzmann's constant and T is the temperature in Kelvins.

In Lord Rayleigh's derivation, the average energy of standing waves did not depend on the wavelength of the standing wave. It is given by

$$\bar{E} = \frac{1}{N}\int_0^\infty E\, f(E)\, dE = kT.$$

Thus, each standing wave mode carries energy $kT$ (on average).

Now that we have expressions for $n$ and $\bar{E}$, we can find the classical prediction for the energy denstiy:

$$u = n \cdot \bar{E} = \frac{8\pi}{\lambda^4} kT.$$

This result is known as the Rayleigh-Jeans formula. It provides a good approximation of thermal radiation at long wavelengths, but dramatically fails at short wavelengths. In fact, it predicts that the radiant intensity should *diverge* as $\lambda \to 0$. This result was called the Ultraviolet Catastrophe since it predicts we should all be blasted by intense ultraviolet and shorter waves. The table below summarizes this derivation and compares it with the quantum derivation proposed by Max Planck.

| Quantity | Classical | Quantum |
|---|---|---|
| # of modes per unit volume | $n = \dfrac{8\pi}{\lambda^4}$ | $n = \dfrac{8\pi}{\lambda^4}$ |
| Mode energy | $E$ (continuous variable) | $E_n = nhf$ (quantized variable) |
| Distribution function | $f(E) \propto e^{-E/kT}$ | $f(E_n) \propto e^{-E_n/kT}$ |
| Ave. energy per mode | $\bar{E} = \dfrac{1}{N}\displaystyle\int_0^\infty E\, f(E)\, dE$ | $\bar{E} = \dfrac{1}{N}\displaystyle\sum_{n=0}^{\infty} E_n f(E_n)$ |
| | $= \dfrac{1}{kT}\displaystyle\int_0^\infty E\, e^{-E/kT}\, dE$ | $= A\displaystyle\sum_{n=1}^{\infty} E_n e^{-E_n/kT}$ |
| | $= kT$ | $= \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}$ |
| Energy density | $u = n \cdot \bar{E}$ | $u = n \cdot \bar{E}$ |
| | $= \left(\dfrac{8\pi}{\lambda^4}\right)(kT)$ | $= \left(\dfrac{8\pi}{\lambda^4}\right)\left(\dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}\right)$ |
| Final Result | $u = \dfrac{8\pi}{\lambda^4} kT$ | $u = \dfrac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1}$ |

### 7.2.2 Quantum Theory of Thermal Radiation

In 1901 Max Planck derived the correct formula for the intensity of thermal radiation by assuming that light was emitted and absorbed in fixed "chunks", each with energy $E = hf$, where

$$h = 6.626 \times 10^{-34} \text{J s}$$

is now known as Planck's constant. It is sometimes useful to express $h$ in units of electron volts:

$$h = 4.136 \times 10^{-15} \text{eV s}.$$

His derivation followed that of Lord Rayleigh, except he assumed that the energy of each standing wave is restricted to a discrete set of *quantized* values $E_n = nhf$. He replaced the classical integral over continuous energies $E$ with a summation over discrete energies $E_n$ (See the Table above). The resulting form of the thermal radiation spectrum is given by

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}.$$

This is known as the Planck function or the Planck distribution. It has been shown to very accurately model the spectral distribution of thermal radiation from radio waves to gamma rays. The Planck function is the first result of quantum physics.

## 7.3 Photons

In 1905 Albert Einstein proposed that in order to explain the photoelectric effect (see below), light must exist as a particle of electromagnetic energy rather than as a wave. This paper, published in the same year (1905) as his theory of special relativity, won him the Nobel Prize in Physics in 1921. It wasn't until 1926 that the term "photon" was used to describe these light particles. The **energy of a photon** is proportional to its frequency:

$$E = hf,$$

where $h$ is the constant that Max Planck introduced 5 years earlier. The energy of a photon may be written in terms of wavelength as

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{\lambda}$$

where $hc = 1240$ eV nm or 1240 MeV fm, where 1 fm $= 10^{-15}$ m. If visible light is defined as having wavelengths between $\lambda = 400$-700 nm, then energy range of visible light photons is $E = 1.77$-3.10 eV, where the low-energy limit (1.77 eV) corresponds to the high-wavelength limit (700 nm).

The following sections describe experiments that support the particle-nature of light, i.e. they give evidence for the existence of photons.

## 7.4 Photoelectric Effect

When light with a sufficiently short wavelength $\lambda$ (or sufficiently high frequency $f$) shines on a metal in a vacuum, the metal is observed to emit electrons. This phenomenon is called the photoelectric effect. Results of the photoelectric effect experiment are as follows:

1. Photoelectrons appear almost instantaneously ($< 10$ns) after the light is turned on
2. Photoelectrons are only emitted when the frequency of the light is above a threshold value $f_c$
3. The maximum kinetic energy $K_{max}$ of the photoelectrons does not depend on the intensity of the radiation. It only depends on the frequency $f$.

Several aspects of this experiment could not be explained using the classical theory for electromagnetic radiation. Albert Einstein showed that the results of the experiment could be completely explained if one assumed that light traveled in tiny "chunks" or particles.

Einstein's assumption was that all of the energy of a given light particle is absorbed by a single electron in the metal. The ejected electrons are sometimes called **photoelectrons** because they are kicked out of the metal due to photons. Thus the photoelectric effect can be written like a chemical equation:

$$\text{photon} + \text{electron} \rightarrow \text{electron}.$$

The behavior of the photoelectric effect is well-described using the photon model of light combined with conservation of energy. We define the work function $\phi$ a metal as the minimum energy required to pull a conduction electron out of it. Thus we imagine the most loosely bound electrons as sitting in an energy level $E = -\phi$ in the metal. The minimum frequency $f_c$ of a photon needed to kick out a photoelectron is thus just $hf_c = \phi$ or

$$f_c = \frac{\phi}{h}.$$

If the photoelectron has leftover kinetic energy $K_{max}$ then the relationship is

$$K_{max} = hf - \phi.$$

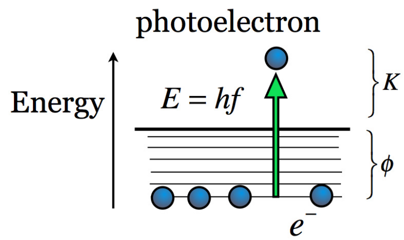


Figure 1: Energy diagram illustrating the photoelectric effect.

### 7.5 X-Ray Production (Bremsstrahlung Radiation)

In some sense, X-ray production is opposite of the photoelectric effect. In the photoelectric effect, a photon strikes a metal to produce an electron. In X-ray production, an electron strikes a metal to produce a photon (often an X-ray photon):

$$\text{electron} \rightarrow \text{electron} + \text{photon}.$$

As the electron encounters the atoms in the metal, it decelerates emitting radiation. This radiation is called Bremsstrahlung radiation, which just means "braking" radiation in German. Because X-ray production is created from electrons with many keV or more of kinetic energy, the work functions of metal in the eV range may be neglected. Applying the photoelectric effect equation gives the minimum wavelength of X-rays

$$\lambda_{min} = \frac{hc}{K}$$

where $K$ is the kinetic energy of the incident electrons.

### 7.6 Pair Production

Pair production is the creation of matter and antimatter particles from the energy of a photon. The energy $E = hf$ of a photon is converted into the rest energy $mc^2$ and kinetic energies $K$ of two

$$\text{photon} \rightarrow \text{particle} + \text{antiparticle}.$$

Like X-ray production, the photon must be in the neighborhood of an atom or other particle that can recoil for momentum to be conserved. The energetics are as follows:

$$hf = E_{particle} + E_{antiparticle} = (mc^2 + K_1) + (mc^2 + K_2).$$

The minimum wavelength to create two particles with mass $m$ occurs when the particles are created at rest (i.e. $K_1 = K_2 = 0$):

$$\lambda_{min} = \frac{hc}{2mc^2} = \frac{1240 \text{ fm MeV}}{2mc^2},$$

where 1 fm $= 10^{-15}$ m. For an electron-positron pair where $mc^2 = 0.511$ MeV, the minimum wavelength is $\lambda_{min} = 1.2$ pm (1 pm $= 10^{-12}$ m). This is a gamma ray.

### 7.7 Annihilation

Annihilation is the opposite of pair production. In pair production, a particle-antiparticle pair is produced from the energy of a photon. In Annihilation, a particle meets its antiparticle, they destroy each other and turn back into energy.

$$\text{particle} + \text{antiparticle} \rightarrow \text{photon} + \text{photon}.$$

In this reverse reaction, two photons must be created if the particle-antiparticle pair meet in the vacuum in order for momentum to be conserved

### 7.8 Compton Scattering

When a photon encounters a nearly free electron it can scatter off it. The interaction is much like two billiard balls undergoing an elastic collision. The basic interaction may be written as

$$\text{photon} + \text{electron} \rightarrow \text{photon} + \text{electron}.$$

Experimentally, one often measures the wavelength of the scattered photon $\lambda'$ as a function of the scattering angle $\theta$ and compares it to the wavelength of the original photon $\lambda$. One can solve for this wavelength shift $\lambda' - \lambda$ by writing down equations of energy and momentum conservation in the relativistic limit:

$$\begin{aligned} E_{before} = E_{after} : & \qquad hf + m_e c^2 = mf' + m_e c^2 + K_e \\ p_{x,before} = p_{x,after} : & \qquad \frac{hf}{c} = \frac{hf'}{c} \cos\theta + p_e \cos\phi \\ p_{y,before} = p_{y,after} : & \qquad 0 = \frac{hf'}{c} \sin\theta - p_e \sin\phi, \end{aligned}$$

where $K_e$ is the kinetic energy of the electron, $p_e$ is the momentum of the electron, $hf/c$ is the initial momentum of the photon and $hf'/c$ is the final momentum of the photon. After a bit of algebra, we can show that the wavelength shift is given by

$$\lambda' - \lambda = \frac{h}{m_e c} \left(1 - \cos\theta\right).$$

The expression $h/m_e c = 0.002426$ nm is called the **Compton wavelength** of the electron. This is a *very* small shift. Its discovery and measurement earned Arthur Compton and Charles Wilson the Nobel Prize in 1927.