Lecture 5. Collisions and Reactions

Contents

Summary. The conversion of energy to mass and mass to energy is evident in high-energy interactions between subatomic particles. We look at inelastic collisions, particle annihilation and pair production.

4.1 Conservation of Momentum and Energy

Just as in Newtonian mechanics, the total momentum $\vec{\mathbf{P}} = \sum \vec{\mathbf{p_i}}$ and energy $E = \sum E_i$ of a system of particles is conserved if no net force acts on the system. These laws are valid in special relativity if the relativistic momentum and total relativistic energy (rest energy plus kinetic energy) are used. The law of **conservation of linear momentum** may be stated as:

> *In an isolated system of particles, the total linear momentum remains constant over time.*

Similarly, the law of **conservation of energy** may be written as:

> *In an isolated system of particles, the total relativistic energy remains constant over time.*

As we will see below, however, mass is *not* conserved in special relativity since mass and energy can be transformed into one another.

4.2 Completely Inelastic Collisions

Completely inelastic collisions occur when two or more particles come together and "stick." The opposite process happens when a single particle splits apart into smaller particles. In Newtonian mechanics, these collisions and reactions are governed by the conservation of momentum and conservation of mass.

To see how relativity changes the rules of inelastic collisions we consider the inelastic collision of two particles with equal masses m (see Figure 2). We pick the reference frame in which particle 1 moves to the right with velocity $v_{1x} = v$ and particle 2 is moves the left with velocity $v_{2x} = -v$. Assume the particles stick together to create a new particle with mass M. The final particle will be at rest because the momentum of the system of initial particles was zero.

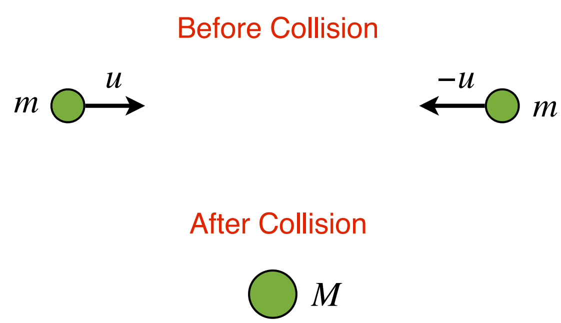

Figure 1: Two particles collide and stick together, creating a more massive particle.

The total energy of the system before the collision is

$$E_{before} = \gamma(v)mc^2 + \gamma(v)mc^2 = 2\gamma(v)mc^2.$$

Because the final particle is at rest, the total energy of the system after the collision is

$$E_{after} = Mc^2$$

Equating these two expressions leads to our desired result:

$$\begin{aligned} E_{before} &= E_{after} \\ 2\gamma(v)mc^2 &= Mc^2 \\ M &= 2\gamma(v)m. \end{aligned}$$

Thus, we see that **mass is not conserved** in this collision! In fact, the mass of the system *increased* as a result of the interaction. Where did the extra mass come from? Let's find the difference between the final mass and the initial mass of the system:

$$\begin{aligned} \Delta m = m_{final} - m_{initial} &= 2\gamma(v)m - 2mc^2 \\ &= 2(\gamma(v) - 1)mc^2 \\ &= 2K. \end{aligned}$$

We see that the increase in mass is equal to the total kinetic energy of the original two particles. Thus, kinetic energy was converted into mass!

Example 1. Suppose two particles each with rest energy 100 MeV and kinetic energy 50 MeV collide head-on and stick to form a new particle. Find the final particle mass.

Solution. The initial energy of the two particles is

$$\begin{aligned} M &= 2mc^2 + 2K \\ &= 2 \cdot 100 \text{ MeV} + 2 \cdot 50 \text{ MeV} = 300 \text{ MeV}. \end{aligned}$$

Because the final particle is at rest, all the energy shows up as rest energy. Thus, the mass of the final particle is 300 MeV/c^2.

4.3 Binding Energy

When two particles are held together by an attractive force, the negative binding energy affects the mass of the system. Consider the case of an electron held onto a proton by the electric force. In classical physics, the kinetic energy K of the electron is $K = -U/2$, where U is the potential energy of the system. For bound orbits, the total energy $E = K + U$ is negative. This total (negative) energy is called the **binding energy**. We denote the binding energy as E_b. Relativistically, we can write the total energy of the system as

$$E = m_1c^2 + m_2c^2 + E_b. \tag{4.10}$$

Equivalently, we can say that the mass of the system is

$$M = E/c^2 = m_1 + m_2 + E_b/c^2. \tag{4.11}$$

Because $E_b < 0$, the mass of the system is less than the mass of the constituent particles that make up the system!

Example 2. Use the fact that the binding energy of the hydrogen atom is -13.6 eV to calculate how much less massive the hydrogen atom is compared to a proton and an electron in isolation.

Solution. The -13.6 eV binding energy corresponds to a mass of

$$\begin{aligned}\Delta m &= -\left(13.6\text{ eV}\right)\frac{1}{c^2}\\ &= -\left(13.6\times 10^{-6}\text{ MeV}\right)\frac{1}{c^2}\left(\frac{1\text{ u}}{931.49\text{ MeV}/c^2}\right)\\ &= -1.46\times 10^{-8}\text{ u}\end{aligned}$$

Because this mass loss is about one hundred millionth the mass of the hydrogen atom ($m_H = 1.0078$ u), it can usually be ignored. This energy is radiated away as an ultraviolet photon when the electron and proton combine to form a neutral hydrogen atom.

Example 3. A deuteron is the nucleus of a deuterium atom. It consists of one proton and one neutron bound together by the strong nuclear force. Find its binding energy by looking up the rest energies of the deuteron, the proton and the neutron.

Solution. The rest energy of a proton is 938.272 MeV/c^2 and that of a neutron is 939.272 MeV/c^2. The sum of the masses is

$$m_p + m_n = 938.272\text{ MeV}/c^2 + 939.272\text{ MeV}/c^2 = 1877.837\text{ MeV}/c^2$$

Since the rest energy of the deuteron is $m_d = 1875.613$ MeV/c^2, the binding energy is

$$E_b = (m_p + m_n)c^2 - m_dc^2 = 1877.837\text{ MeV}/c^2 - 1875.613\text{MeV}/c^2 = 2.224\text{MeV}/c^2.$$

When a neutron and proton combine to form a deuteron, 2.224 MeV/c of energy is released. We see that this nuclear reaction releases more than 10^5 times more energy than the "chemical" reaction in the previous example.

4.4 Weighing a System

In classical mechanics we are used to the idea that mass is conserved. The mass of a system of particles equals the sum of the masses of the component particles, i.e.

$$M = \sum m_i \qquad \text{(Classical)}$$

However, relativity tells us that if we place a system of particles on a scale, the mass we measure equals the sum of the particle masses plus the equivalent mass of all energies (kinetic and potential) that might be present.

$$M = \sum m_i + \sum K_i/c^2 + \sum U_i/c^2 \qquad \text{(Relativity)}$$

This means that when we find the mass of any object with internal structure, we generally have no way of separating out how much of the mass may be due to the mass of the constituent particles and how much may be do to the kinetic or potential energies.

When two particles come together to form a new particle (or a new system), several scenarios are theoretically possible:

1. The two initial particles undergo a nuclear transformation to form a new fundamental particle. One can think of the initial masses being converted to energy, and then a new particle forms from that energy.

2. If the final particle is a *composite* particle, we could visualize an imaginary spring between the two initial masses. As the particles approach each other, they compress the spring. At the point of maximum compression, the initial kinetic energy is converted to potential energy of the spring. If the spring is "locked" in place by, say the nuclear force, then **this potential energy increases the mass of the system**.

3. If the two composite particles are bound by an attractive force, the corresponding potential energy will be negative. **The binding energy decreases the mass of the system**.

4. A final possibility is that the two initial particles somehow attach to each other, i.e. "hold hands," and orbit around one another. In this case, the kinetic energy of the orbiting particles is seen by an outside observer as increased mass.

4.5 Antimatter

All fundamental particles found in nature have an anti-matter "partner." Antimatter particles have the same mass but opposite electrical charge. For example, the anti-proton $\bar{p}$ has a negative charge $-e$ and a mass of 938 MeV/c^2. The anti-neutron $\bar{n}$ is neutral and the anti-electron e^+ (also called the positron) is positive. Antimatter is actually a consequence of special relativity, although the details are beyond the scope of this course.

4.6 Particle Annihilation

When a particle and its antiparticle meet, they mutually destroy each other and their masses are converted into energy. For example if an electron e^- meets a positron e^+ they annihilate and turn into energy. We can write the reaction like this:

$$e^- + e^+ \to \gamma,$$

where γ is the symbol for a gamma ray. Assuming the electron and positron had negligible kinetic energy before the collision, we can write the energy of the gamma ray as

$$E = (2)(0.511\text{MeV}) = 1.022\text{MeV}.$$

This energy will be produced if the two particles are approximately at rest.

4.7 Pair Production

Pair production is the inverse process to particle annihilation. In pair production, a particle-antiparticle pair is created from energy. Symbolically it may be represented

$$\gamma \to e^- + e^+.$$

Example 4. Please see Krane, example 2.18.
