Homework 1

Goal: To learn to use Matlab as a calculator for simple computations involving scalars, vectors and matrices.

Due: Feb. 12

Read and work through the calculations in Spencer and Ware, Chapter 1. The following problems are organized by the section number. After working through each section, use Matlab to answer each of the following problems. You just need to turn in the attached answer sheet for this homework. Ask the instructor to sign off on each problem. If you find another student who has been signed off, you can ask them to sign.

Section 1.2

1. Use Matlab to calculate the following to 15 decimal places:
   (a) π
   (b) $\frac{1}{\sqrt{2}}$
2. Use Matlab to calculate the following to 5 decimal places:
   (a) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$
   (b) $1+\frac{1}{1+\frac{1}{1+\frac{1}{2}}}$
3. Estimate how large a number Matlab can handle. Try typing in, for example, `1e100` to represent 1×10^{100}. If the number is too large, Matlab will just return `Inf`. Find the largest value Matlab can handle to the nearest order of magnitude. (You can try googling the answer if you wish, but then verify it yourself).

Section 1.4

4. In this problem you will use Matlab to do some simple relativistic calculations. Consider the following particle speeds: $v = 0.9999c$, $v = 0.99c$, $v = 0.9c$, $v = 0.1c$, $v = 10^{-4}c$.
   (a) Create an array to store the velocities. To make things simple, you can define a dimensionless velocity $\beta = v/c$. The dimensionless velocities are then $\beta = 0.9999$, $\beta = 0.99$, etc.
   (b) Calculate the gamma factor $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ for each velocity and store the result in an array. In terms of the dimensionless velocity, the gamma factor is $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.
   (c) Calculate the percentage of the particle's total energy that is stored as kinetic energy:
   $$\%KE = \left|\frac{KE}{E}\right| \times 100\% = \left|\frac{(\gamma-1)mc^2}{\gamma mc^2}\right| \times 100\% = \left|\frac{\gamma-1}{\gamma}\right| \times 100\%.$$
   (d) Does your result make sense? Why?
5. Calculate the sum of all the integers between (and including) 1 and 100. Repeat this calculation for integers up to 1000 and then 10,000. How does changing the upper limit affect your result?
6. Use the colon operator to create a vector A containing all the odd numbers between 2 and 50.
   (a) Calculate the sum of all the elements of A.
   (b) Add the values of the first and last elements of A.

(c) Find the number of elements in A.

(d) What is the value of the middle element of A?

7. Define the following vectors and matrices:

$$x = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \qquad y = \begin{pmatrix} 6 & 7 & 3 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Use Matlab to calculate the following:

(a) $\mathbf{yx}$

(b) $\mathbf{xy}$

(c) calculate the row vector $\mathbf{z}$ whose elements are the products $\mathbf{z_i} = \mathbf{x_i y_i}$

(d) calculate the column vector $\mathbf{z}$ whose elements are the products $\mathbf{z_i} = \mathbf{x_i y_i}$

(e) calculate the vector $\mathbf{z}$ whose elements are the products $\mathbf{z_i} = \mathbf{x_i^3}$

(f) $\mathbf{A}^\intercal$

(g) numerically verify that $\mathbf{AB} \neq \mathbf{BA}$

8. Variables may be saved in `.mat` files to be used at a later time. Download the file `constants.mat` from the course website. Place the file in your working Matlab directory. Clear all the variables from memory using the `clear` command, then load in the `constants.mat` file using the command `load 'constants'`. You should now see a list of fundamental constants (in MKS units) in the workspace window to the right of the command window. Use these variables to calculate the following:

(a) Ground-state energy of the hydrogen atom (in electron volts): $E_0 = \frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}$

(b) Bohr radius of the hydrogen atom (in nanometers): $a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2}$

The file `constants.mat` contains the following variables (defined in MKS units):

- `G = Gravitational constant`
- `c = speed of light`
- `e = fundamental charge`
- `eV = electron volt (in Joules)`
- `eps0 = permittivity of free space`
- `h = Planck's constant`
- `hbar = Planck's constant / 2 pi`
- `k = Boltzmann constant`
- `me = mass of electron`
- `mn = mass of neutron`
- `mp = mass of proton`
- `mu0 = permeability of free space`
- `u = atomic mass unit`