Homework 7
Due Tuesday, Oct. 30

1. For each of the following complex numbers, find the magnitude, direction, real part, and imaginary part and then sketch it in the complex plane.

   a) $1 + 3i$

   b) $\dfrac{1}{1 + 3i}$

   c) $e^{i\phi}$

   d) $e^{i\phi}(1 - i)$

   e) $e^{i\pi/4}(2 + i)$

2. Find the complex conjugate of each of the numbers in problem 1, and then repeat the calculations in problem 1 for each (i.e. recalculate the magnitude, direction, etc. and sketch).

3. Given two complex numbers $z_1 = |z_1| e^{i\theta_1}$ and $z_2 = |z_2| e^{i\theta_2}$, calculate $\dfrac{z_2}{z_1}$. What affect does dividing one complex number by another have on the resultant modulus and polar angle?

4. Use Euler's formula $e^{i\phi} = \cos\phi + i\sin\phi$ and $e^{-i\phi} = \cos\phi - i\sin\phi$ to derive the following:

   a) $\cos\phi = \dfrac{e^{i\phi} + e^{-i\phi}}{2}$

   b) $\sin\phi = \dfrac{e^{i\phi} - e^{-i\phi}}{2i}$

5. Substitute $\phi = \alpha + \beta$ into Euler's formula to derive the following trig identities:

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

6. Use complex notation to show that a standing wave can be created by adding two oppositely moving traveling waves.

7. Compare the following complex traveling waves. Which move in the +x direction? Which move in the –x direction? Imagine holding x constant. Which circulate clockwise/counterclockwise in the complex plane as t increases? Now hold t constant. Which circulate clockwise/counterclockwise in the complex plane as x increases? You may want to make a table to organize your results.

   a) $\psi(x, t) = e^{i(kx - \omega t)}$
   b) $\psi(x, t) = e^{i(\omega t - kx)}$
   c) $\psi(x, t) = e^{i(kx + \omega t)}$
   d) $\psi(x, t) = e^{-i(kx + \omega t)}$

8. Consider complex functions of the form $\Psi(x,t) = \psi(x)e^{i\omega t}$, where different $\psi(x)$ functions are given below. Use Mathematica to animate each wave. Describe the motion of each as a function of time. Which are traveling waves? Which are standing waves? Which are wave packets? Which are something else? What general conclusions (if any) can you draw about the behavior of these functions?
   a) $\psi(x) = e^{ikx}$
   b) $\psi(x) = \cos kx$
   c) $\psi(x) = i\cos kx$
   d) $\psi(x) = e^{ik_1x} + e^{ik_2x}$
   e) $\psi(x) = e^{-x^2/a^2}$
   f) $\psi(x) = e^{-x^2/a^2}\cos kx$
   g) $\psi(x) = e^{-x^2/a^2}e^{ikx}$
   h) $\psi(x) = \cos^2 kx + i\sin^2 kx$ (just for fun)

   When plotting these functions, you might start by setting the domain to be $-10 < x < 10$ and the range to be $-2 < y < 2$. Set the angular frequency to be around $\omega = 3$, the wave numbers to around $k = 2$, and the Gaussian parameter $a \approx 3$. Feel free to play with these parameters to see how they affect the wave shape and evolution.