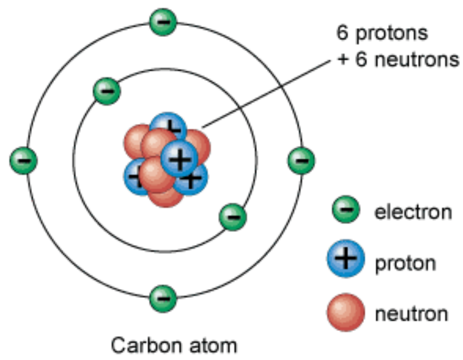
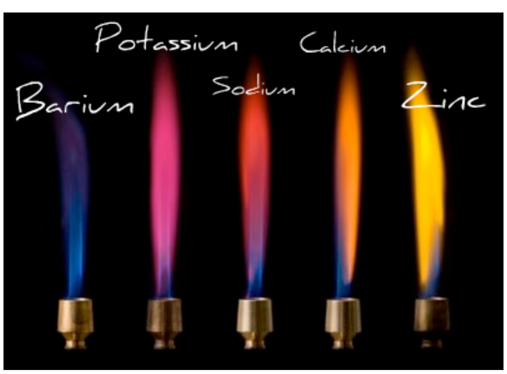
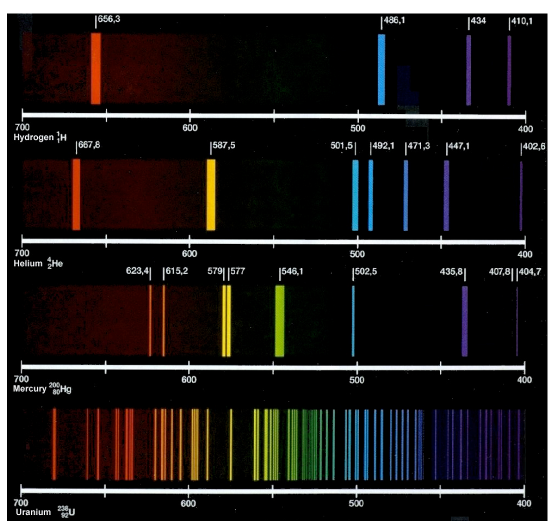
Chapter 6: Rutherford-Bohr Model of Atom

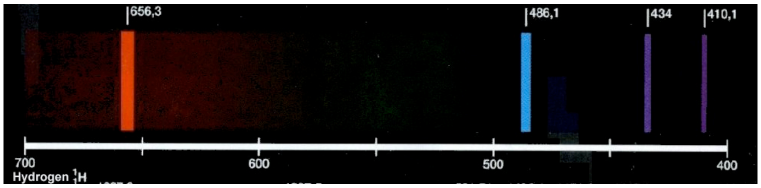


Emission line spectra of flames





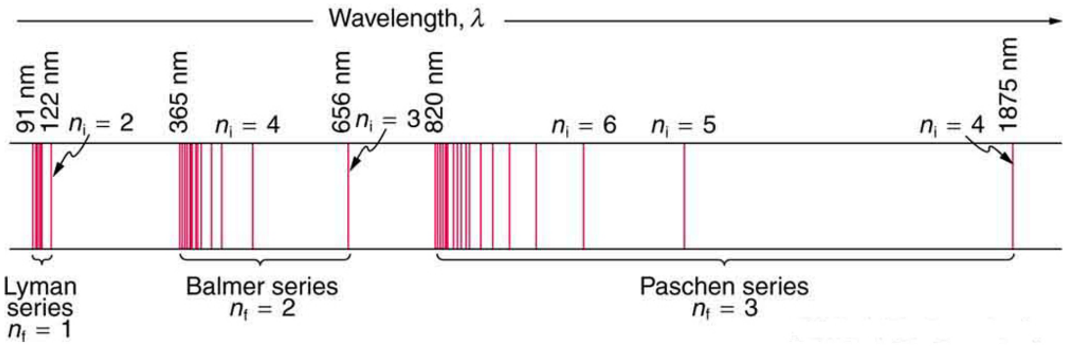
Hydrogen Spectrum - Balmer Lines (1885)



$$H_\alpha = 656.3 \text{ nm}$$
$$H_\beta = 486.1 \text{ nm}$$
$$H_\gamma = 434.0 \text{ nm}$$
$$H_\delta = 410.2 \text{ nm}$$

$$\lambda = (656.3 \text{ nm})\frac{n^2}{n^2-4} \qquad n = 3,4,5...$$

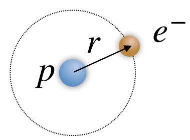
Hydrogen Spectrum



$$\lambda = \lambda_{\text{limit}}\frac{n^2}{n^2-n_0^2} \qquad n = n_0+1, n_0+2, n_0+3, ...$$

Bohr Atom

Semi-classical, semi-quantum model of atom



$$F_c = F_e$$

$$m\frac{v^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$$

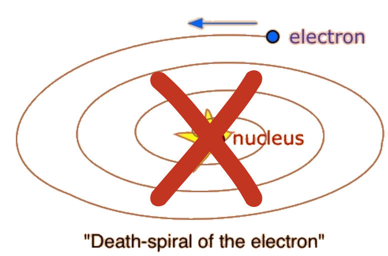
Kinetic Energy $K = \frac{1}{2}mv^2 = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r}$

Potential Energy $U = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$

Total Energy $E = K + U = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r}$

Bohr Atom

Bohr's assumption: some electron orbits are stationary and don't produce electromagnetic radiation.



These "allowed" orbits have angular momentum values equal to

$$L = mvr = n\hbar \qquad n = 1,2,3...$$

Substitute and solve for the orbital radius:

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2}n^2 = a_0 n^2$$

where the Bohr radius is defined to be $a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.0529$ nm

Bohr Atom

Allowed energy levels of the Bohr atom are found by plugging into the energy equation:

$$E = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r} \qquad r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2}n^2 = a_0 n^2$$

Energy levels:

$$\boxed{E_n = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = \frac{-13.6 \text{ eV}}{n^2} \qquad n = 1,2,3,\ldots}$$

Ground state = $E_1 = -13.6 \text{ eV}$

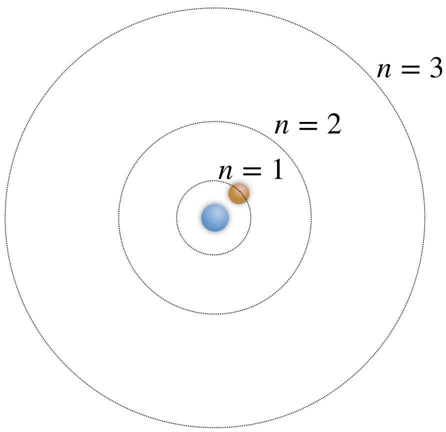
Excited states = $E_2, E_3, E_4, \ldots$

Problem: Find the quantum number n such that the orbital radius = 1 mm

Bohr Atom

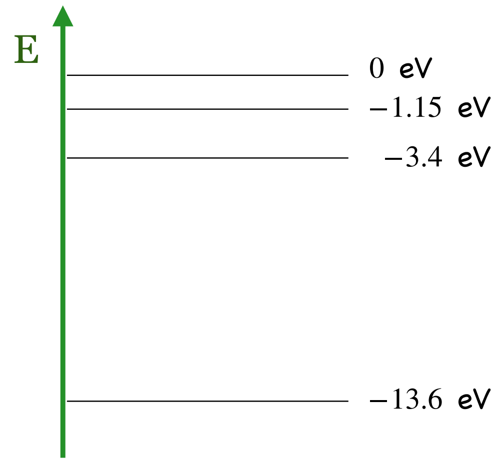
Orbits

$$r_n = a_0 n^2$$



Energy Level Diagram

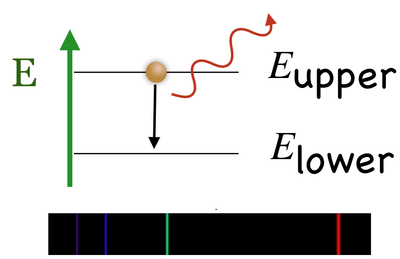
$$E_n = \frac{-13.6\ \text{eV}}{n^2}$$



Atomic Transitions

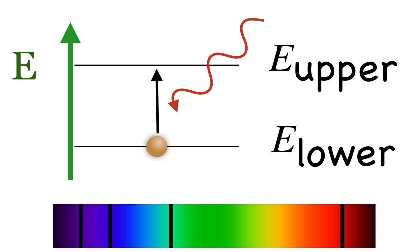
Emission - An electron can jump down to a lower orbit by radiating a photon with energy

$$hf = E_{\text{upper}} - E_{\text{lower}}$$



Absorption - An electron can jump up to a higher orbit by absorbing a photon with energy

$$hf = E_{\text{upper}} - E_{\text{lower}}$$

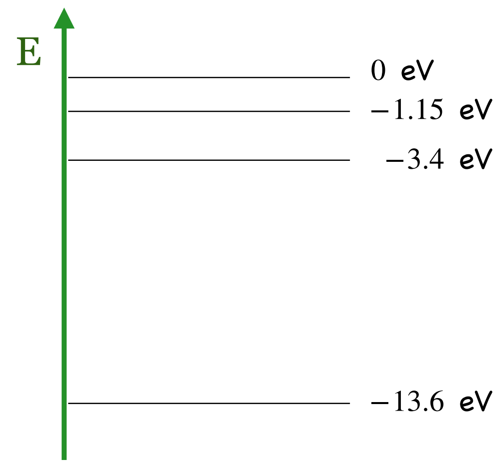


Hydrogen Transitions

Energy of photons emitted or absorbed by hydrogen

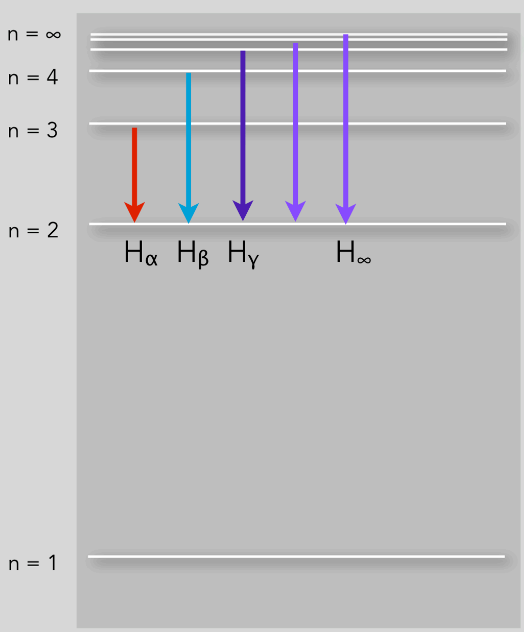
$$hf = E_{n_u} - E_{n_l} = \frac{-13.6\ \text{eV}}{n_u^2} - \frac{-13.6\ \text{eV}}{n_l^2}$$

$$\frac{1}{\lambda} = = \frac{13.6\ \text{eV}}{hc}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$



Find: wavelength of the n=2 to n=1 transition

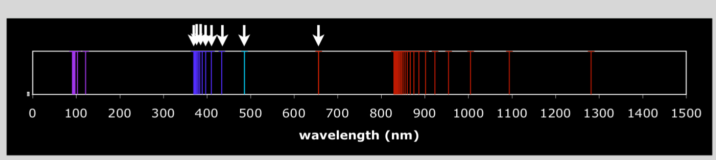
Balmer Series



Transitions to n = 2 state from higher state.

$$h\nu = E_{high} - E_{low} = 13.6\ \text{eV}\left(\frac{1}{2^2} - \frac{1}{n_{high}^2}\right)$$

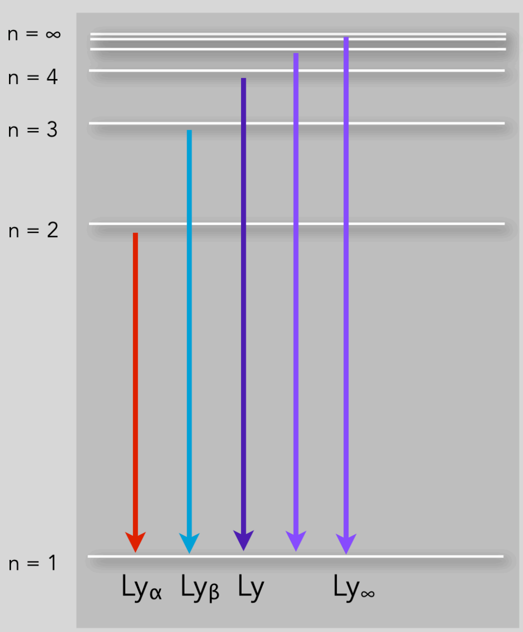
H_α: n = 3 ➞ 2	λ = 656.3 nm	
H_β: n = 4 ➞ 2	λ = 486.1 nm	
H_γ: n = 5 ➞ 2	λ = 434.0 nm	
. . .		
H_∞: n = ∞ ➞ 2	λ = 364.6 nm	Balmer Limit



Calculate the wavelength of the n = 6 to 2 transition in hydrogen.

$$\frac{1}{\lambda} = = \frac{13.6\ \text{eV}}{hc}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

Lyman Series



Transitions to n = 1 state from higher state.

$$h\nu = E_{high} - E_{low} = 13.6\ \text{eV}\left(\frac{1}{1^2} - \frac{1}{n_{high}^2}\right)$$

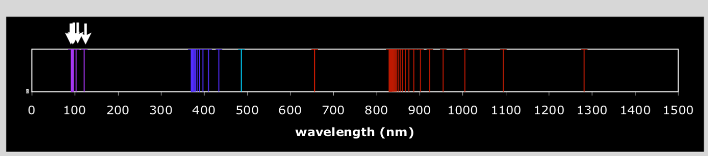
L_α: n = 2 → 1 λ = 121.57 nm

L_β: n = 3 → 1 λ = 102.57 nm

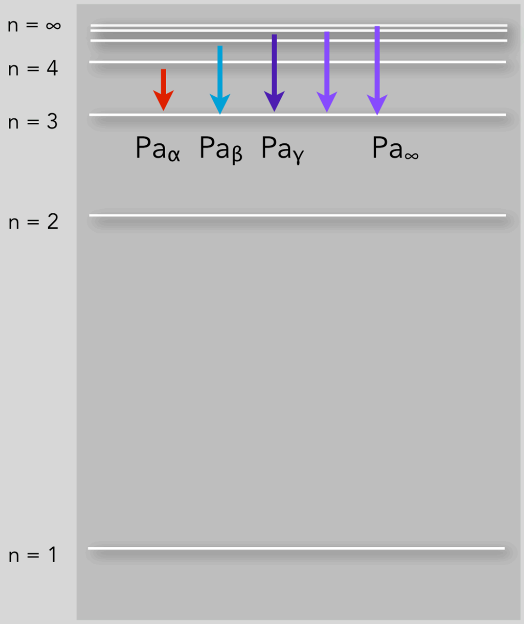
L_γ: n = 4 → 1 λ = 97.24 nm

. . .

L_∞: n = ∞ → 1 λ = 91.18 nm Lyman Limit



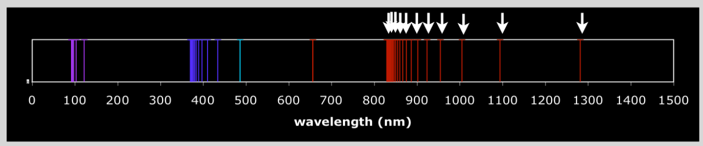
Paschen Series



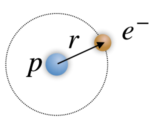
Transitions to n = 3 state from higher state.

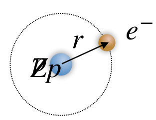
$$h\nu = E_{high} - E_{low} = 13.6\ \mathrm{eV}\left(\frac{1}{3^2} - \frac{1}{n_{high}^2}\right)$$

Paα: n = 4 → 3 λ = 1875.1 nm
Paβ: n = 5 → 3 λ = 1281.8 nm
Paγ: n = 6 → 3 λ = 1093.8 nm
. . .
Pa∞: n = ∞ → 3 λ = 820.4 nm Paschen Limit



Problem: Derive the energy levels for "hydrgenic" ions that have Z protons and 1 electron





$$m\frac{v^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \qquad m\frac{v^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$$

$$E = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r} \qquad E = -\frac{1}{8\pi\epsilon_0}\frac{Ze^2}{r}$$

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2}n^2 = a_0 n^2 \qquad r_n = \frac{4\pi\epsilon_0\hbar^2}{mZe^2}n^2 = \frac{a_0}{Z}n^2$$

$$E_n = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = \frac{-13.6\ \mathrm{eV}}{n^2} \qquad E_n = -\frac{mZ^2e^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = (-13.6\ \mathrm{eV})\frac{Z^2}{n^2}$$

Ionization Energy of Hydrogenic Ions

The ionization energy is the energy needed to ionize an atom from a particular excitation state.

$$\chi_n = (13.6 \text{ eV})\frac{Z^2}{n^2}$$

Homework problem 29. Show that Bohr's assumption that angular momentum is quantized is equivalent to saying that an integer number of de Broglie waves wrap around the atom.

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Homework problem 28. Derive the Bohr radius for an atom in which the electron is held onto the proton by the gravitational force in stead of the electric force.

Find the photon energy for transition from n = 2 to n = 1 orbits.